

JEE-MAIN EXAMINATION – JANUARY 2026

(HELD ON THURSDAY 22nd JANUARY 2026)

TIME : 3:00 PM TO 06:00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let n be the number obtained on rolling a fair die. If the probability that the system
 $x - ny + z = 6$
 $x + (n-2)y + (n+1)z = 8$
 $(n-1)y + z = 1$

Has a unique solution is $\frac{k}{6}$, then the sum of k and all possible values of n is :

- (1) 21 (2) 24
 (3) 20 (4) 22

Ans. (4)

Sol. $x - ny + z = 6$
 $x + (n-2)y + (n+1)z = 8$
 $(n-1)y + z = 1$

$$\begin{vmatrix} 1 & -n & 1 \\ 1 & (n-2) & n+1 \\ 0 & n-1 & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow n^2 - 3n + 2 \neq 0$$

$$n \neq 1, 2$$

for unique solution $n = 3, 4, 5, 6$

Now

P (probability when system of equations has unique solution) = $\frac{4}{6}$

$$\text{So } k = 4$$

$$\text{Now required sum} = 4 + (3 + 4 + 5 + 6) = 22$$

2. If the mean deviation about the median of the numbers $k, 2k, 3k, \dots, 1000k$ is 500, then k^2 is equal to :

- (1) 16 (2) 4
 (3) 1 (4) 9

Ans. (2)

$$\text{Sol. } \because \text{median} = \frac{1001k}{2} = X_M$$

$$\therefore \text{mean deviation about median} = \frac{\sum |X_i - X_M|}{n}$$

$$= \frac{2 \left(\frac{k}{2} + \frac{3k}{2} + \frac{5k}{2} + \dots 500 \text{ terms} \right)}{1000}$$

$$= \frac{2 \cdot \frac{k}{2} (500)^2}{1000} = \frac{500k}{2} = 500 \text{ (given)}$$

$$\therefore k = 2$$

$$\therefore k^2 = 4$$

3. The number of elements in the relation $R = \{(x, y) : 4x^2 + y^2 < 52, x, y \in \mathbb{Z}\}$ is

- (1) 77 (2) 89
 (3) 67 (4) 86

Ans. (1)

Sol. $4x^2 + y^2 < 52, x, y \in \mathbb{Z}$

$$\begin{array}{ll} \downarrow & \downarrow \\ 0 & 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7 \rightarrow 1 \times 15 = 15 \\ \pm 1 & 0, \pm 1, \pm 2, \pm 3, \dots, \pm 6 \rightarrow 2 \times 13 = 26 \\ \pm 2 & 0, \pm 1, \pm 2, \pm 3, \dots, \pm 5 \rightarrow 2 \times 11 = 22 \\ \pm 3 & 0, \pm 1, \pm 2, \pm 3 \rightarrow 2 \times 7 = 14 \end{array}$$

Number of elements = 77

4. Let $S = \{z \in \mathbb{C} : 4z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} |z|^2$ is equal to :

- (1) $\frac{3}{16}$ (2) $\frac{7}{64}$
 (3) $\frac{1}{16}$ (4) $\frac{5}{64}$

Ans. (1)

Sol. $4z^2 + \bar{z} = 0$

$$\text{let } z = x + iy$$

$$4(x + iy)^2 + x - iy = 0$$

$$4x^2 - 4y^2 + 8xyi + x - iy = 0$$

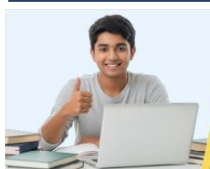
$$4x^2 - 4y^2 + x = 0 \text{ \& } y(8x - 1) = 0$$

$$\Rightarrow y = 0 \text{ or } x = \frac{1}{8}$$

$$\text{If } y = 0, 4x^2 + x = 0$$

$$x = 0, \frac{-1}{4}$$

$$\therefore z_1 = 0 + 0.i \quad |z_1|^2 = 0$$



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$$z_2 = 0 - \frac{1}{4}i \quad |z_2|^2 = \frac{1}{16}$$

$$\text{If } x = \frac{1}{8},$$

$$4 \times \frac{1}{64} - 4y^2 + \frac{1}{8} = 0$$

$$\Rightarrow 4y^2 = \frac{3}{16} \Rightarrow y = \pm \frac{\sqrt{3}}{8}$$

$$\therefore z_3 = \frac{1}{8} + \frac{\sqrt{3}}{8}i \quad |z_3|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$$

$$z_4 = \frac{1}{8} - \frac{\sqrt{3}}{8}i \quad |z_4|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$$

$$\therefore \sum_{i=1}^n |z_i|^2 = 0 + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$$

5. If $\lim_{x \rightarrow 0} \frac{e^{(a-1)x} + 2 \cos bx + (c-2)e^{-x}}{x \cos x - \log_e(1+x)} = 2$, then $a^2 +$

$b^2 + c^2$ is equal to :

(1) 5 (2) 3

(3) 7 (4) 9

Ans. (3)

Sol. $\lim_{x \rightarrow 0} \frac{\left(1 + (a-1)x + \frac{(a-1)^2 x^2}{2!}\right) + 2\left(1 - \frac{b^2 x^2}{2!}\right) + (c-2)\left(1 - x + \frac{x^2}{2!}\right)}{x\left(1 - \frac{x^2}{2!}\right) - \left(x - \frac{x^2}{2} + \dots\right)} = 2$

$$\lim_{x \rightarrow 0} \frac{(1+2+c-2) + x(a-1-c+2) + x^2\left(\frac{(a-1)^2}{2} - b^2 + \frac{(c-2)}{2}\right)}{\frac{x^2}{2} - \frac{x^3}{2!} + \dots} = 2$$

For which

$$\therefore c + 1 = 0 \Rightarrow c = -1$$

$$\therefore a - c = -1 \Rightarrow a = -2$$

$$\therefore \frac{(a-1)^2}{2} - b^2 + \left(\frac{c-2}{2}\right) = 1$$

$$\frac{9}{2} - b^2 - \frac{3}{2} = 1 \Rightarrow b^2 = 2$$

$$a^2 + b^2 + c^2 = 4 + 2 + 1 = 7$$

6. If $y = y(x)$ satisfies the differential equation

$$16\left(\sqrt{x+9\sqrt{x}}\right)\left(4+\sqrt{9+\sqrt{x}}\right)\cos y \, dy = (1+2$$

$$\sin y)dx, x > 0 \text{ and } y(256) = \frac{\pi}{2}, y(49) = \alpha, \text{ then } 2$$

$\sin \alpha$ is equal to :

(1) $2\sqrt{2}-1$ (2) $2(\sqrt{2}-1)$

(3) $3(\sqrt{2}-1)$ (4) $\sqrt{2}-1$

Ans. (1)

Sol. $\int \frac{\cos y}{1+2\sin y} dy = \int \frac{dx}{16\left(\sqrt{9\sqrt{x}+x}\right)\left(4+\sqrt{9+\sqrt{x}}\right)}$

$$4 + \sqrt{9 + \sqrt{x}} = t$$

$$\frac{1}{2\sqrt{9+\sqrt{x}}} \times \frac{dx}{2\sqrt{x}} = 1dx$$

$$\frac{1}{2} \ln |1+2\sin y| = \int \frac{4dt}{16t} + C$$

$$\frac{1}{2} \ln |1+2\sin y| = \frac{1}{4} \ln |4 + \sqrt{9 + \sqrt{x}}| + C$$

$$\frac{1}{2} \ln(2\sin y + 1) = \frac{1}{4} \ln |4 + \sqrt{9 + \sqrt{x}}| + C$$

Substituting $\left(256, \frac{\pi}{2}\right)$

$$\frac{1}{2} \ln 3 = \frac{1}{4} \ln 3 + C \quad C = 0$$

Substituting $(49, \alpha)$

$$\frac{1}{2} \ln(2\sin \alpha + 1) = \frac{1}{4} \ln 8$$

$$\ln(2\sin \alpha + 1) = \frac{1}{2} \ln 8$$

$$\ln(2\sin \alpha + 1) = \ln 2\sqrt{2}$$

$$2\sin \alpha + 1 = 2\sqrt{2}$$

$$2\sin \alpha = 2\sqrt{2} - 1$$



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7. Among the statements

(S1) : If $A(5, -1)$ and $B(-2, 3)$ are two vertices of a triangle, whose orthocentre is $(0, 0)$, then its third vertex is $(-4, -7)$ and

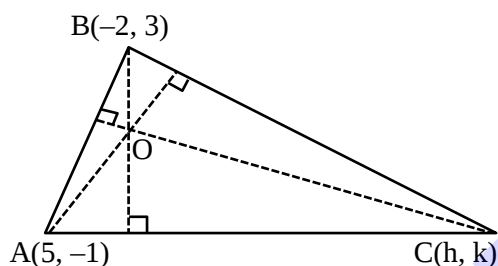
(S2) : If positive numbers $2a, b, c$ are three consecutive terms of an A.P., then the lines $ax + by + c = 0$ are concurrent at $(2, -2)$,

- (1) Only (S1) is correct
(2) Only (S2) is correct
(3) Both are incorrect
(4) Both are correct

Ans. (4)

Sol. Solution of statement-1

$$m_{AO} \cdot m_{BC} = -1$$



$$\Rightarrow 5h - k + 13 = 0 \quad \dots(1)$$

$$\& m_{AB} \cdot m_{OC} = -1$$

$$\Rightarrow 4k = 7h \quad \dots(2)$$

$$\Rightarrow \text{third vertex is } (-4, -7)$$

$\therefore$ Statement 1 is correct.

Solution of statement-2

$2a, b, c \rightarrow \text{A.P.}$

$$b = \frac{2a + c}{2}$$

$$\Rightarrow 2a - 2b + c = 0$$

$\therefore$ lines $ax + by + c = 0$ are concurrent then

$$\frac{x}{2} = \frac{y}{-2} = \frac{1}{1}$$

$$x = 2 \text{ and } y = -2$$

$\therefore$ Point of concurrency is $(2, -2)$

$\therefore$ Statement 2 is correct.

8. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{j} + 2\hat{k}$, $\lambda \in \mathbb{Z}$ be two vectors, Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz -plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $(\vec{c} \cdot \vec{d})^2$ is equal to :

- (1) 26 (2) 104
(3) 208 (4) 52

Ans. (3)

$$\text{Sol. } \vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \lambda\hat{j} + 2\hat{k}; \lambda \in \mathbb{Z}$$

$$\vec{c} = \vec{a} \times \vec{b} = (-2 - \lambda)\hat{i} - 4\hat{j} + 2\lambda\hat{k}$$

$$|\vec{c}| = \sqrt{53}$$

$$\Rightarrow 5\lambda^2 + 4\lambda - 33 = 0$$

$$\lambda = 2.2 \text{ or } -3$$

$$\Rightarrow \boxed{\lambda = -3}$$

$$\vec{c} = \hat{i} - 4\hat{j} - 6\hat{k}$$

$$\text{let } \vec{d} = y\hat{j} + z\hat{k}$$

$$|\vec{d}| = 2$$

$$\Rightarrow y^2 + z^2 = 4$$

$$(\vec{c} \cdot \vec{d})^2 = (4y + 6z)^2 \leq (\sqrt{4^2 + 6^2} \times \sqrt{y^2 + z^2})^2 \leq 208$$

9. If $X = \begin{bmatrix} x \\ x \\ z \end{bmatrix}$ is a solution of the system of equations

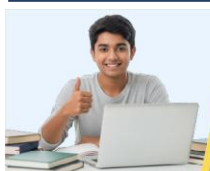
$$AX = B, \text{ where } \text{adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \text{ then } |x + y + z| \text{ is equal to :}$$

- (1) 3 (2) $\frac{3}{2}$
(3) 1 (4) 2

Ans. (4)

$$\text{Sol. } X = A^{-1}B = \left(\frac{\text{adj } A}{|A|} \right) B$$



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$$= \pm \frac{1}{10} \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$$

$$= \pm \frac{1}{10} \begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix} = \pm \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\therefore |x + y + z| = 2$$

10. Let L be the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+3}{6}$ and let S be the set of all points (a, b, c) on L, whose distance from the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+9}{0}$ along the line L is 7. Then $\sum_{(a,b,c) \in S} (a+b+c)$ is equal to :

- (1) 34 (2) 28
(3) 40 (4) 6

Ans. (1)

Sol. M is the point of intersection of L_1 & L_2

$$\Rightarrow 2\lambda - 1 = 2\mu - 1, 3\lambda - 1 = 3\mu - 1, 6\lambda - 3 = 9$$

$$\Rightarrow \lambda = 2 = \mu$$

$$\Rightarrow M(3, 5, 9)$$

Now let point P be $(2K - 1, 3K - 1, 6K - 3)$ on L_2 such that $PM = 7$

$$\Rightarrow \sqrt{(2K-4)^2 + (3K-6)^2 + (6K-12)^2} = 7$$

$$\Rightarrow 49K^2 + 196 - 196K = 49$$

$$\Rightarrow K^2 + 4 - 4K = 1$$

$$\Rightarrow K^2 - 4K + 3 = 0$$

$$\Rightarrow K = 1, 3$$

So points P & Q are (1, 2, 3) & (5, 8, 15)

So sum of all co-ordinates of P & Q = 34

11. Let P $(10, 2\sqrt{15})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S'. If the length of its latus rectum is 8, then the square of the area of $\Delta PSS'$ is equal to :

- (1) 4200 (2) 900
(3) 1462 (4) 2700

Ans. (4)

Sol. P $(10, 2\sqrt{15})$ lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\therefore \frac{100}{a^2} - \frac{60}{b^2} = 1 \quad \dots(1)$$

$$\therefore \text{length of latus rectum} = 8$$

$$\frac{2b^2}{a} = 8 \Rightarrow \frac{b^2}{a} = 4 \quad \dots(2)$$

From (1) & (2)

$$\frac{100}{a^2} - \frac{60}{4a} = 1$$

$$400 - 60a = 4a^2$$

$$4a^2 + 60a - 400 = 0$$

$$a^2 + 15a - 100 = 0$$

$$a = 5 \text{ \& } -20 \text{ (rejected)}$$

$$\Rightarrow b = \sqrt{20}$$

$$\therefore \text{Hyperbola is } \frac{x^2}{25} - \frac{y^2}{20} = 1$$

$$\therefore \text{Focal length } S_1S_2 = 2ae = 2.5 \cdot \left(\sqrt{1 + \frac{4}{5}} \right) = 6\sqrt{5}$$

$$\therefore \text{Area of } \Delta PS_1S_2 = \frac{1}{2} \cdot 6\sqrt{5} \cdot 2\sqrt{15} = 30\sqrt{3} = A$$

$$\therefore A^2 = 2700$$

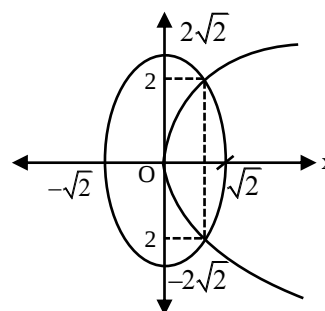
12. The area of the region

$$A = \{(x, y) : 4x^2 + y^2 \leq 8 \text{ and } y^2 \leq 4x\} \text{ is :}$$

- (1) $\frac{\pi}{2} + 2$ (2) $\pi + \frac{2}{3}$
(3) $\pi + 4$ (4) $\frac{\pi}{2} + \frac{1}{3}$

Ans. (2)

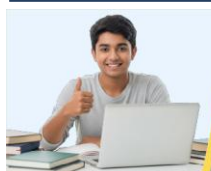
Sol.



$$A = \int_0^2 2\sqrt{x} dx + 2 \int_1^{\sqrt{2}} \sqrt{8-4x^2} dx$$

$$= \frac{8}{3} \left(x^{\frac{3}{2}} \right) \Big|_0^1 + 4 \int_1^{\sqrt{2}} \sqrt{2-x^2} dx$$

$$= \frac{8}{3} + 4 \times \frac{1}{2} \left[x\sqrt{2-x^2} + 2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \right] \Big|_1^{\sqrt{2}}$$



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$$= \frac{8}{3} + 2 \left[2 \times \frac{\pi}{2} - 1 - 2 \times \frac{\pi}{4} \right]$$

$$= \frac{8}{3} + 2\pi - 2 - \pi = \pi + \frac{2}{3} \text{ sq. units}$$

13. Let α, β be the roots of the quadratic equation

$$12x^2 - 20x + 3\lambda = 0, \lambda \in \mathbb{Z}. \text{ If } \frac{1}{2} \leq |\beta - \alpha| \leq \frac{3}{2},$$

then the sum of all possible values of λ is :

- (1) 6 (2) 1
(3) 3 (4) 4

Ans. (3)

Sol. $\frac{1}{2} \leq |\alpha - \beta| \leq \frac{3}{2}$

$$\frac{1}{4} \leq |\alpha - \beta|^2 \leq \frac{9}{4}$$

$$\frac{1}{4} \leq (\alpha + \beta)^2 - 4\alpha\beta \leq \frac{9}{4}$$

$$\frac{1}{4} \leq \frac{25}{9} - 4 \times \frac{\lambda}{4} \leq \frac{9}{4}$$

$$-\frac{91}{36} \leq -\lambda \leq \frac{-19}{36}$$

$$\frac{19}{36} \leq \lambda \leq \frac{91}{36}$$

$$\lambda = 1, 2$$

$$\text{Sum} = 3$$

14. Let the domain of the function

$$f(x) = \log_3 \log_5 (7 - \log_2 (x^2 - 10x + 85)) + \sin^{-1} \left(\left| \frac{3x-7}{17-x} \right| \right)$$

be $(\alpha, \beta]$. Then $\alpha + \beta$ is equal to :

- (1) 10 (2) 12
(3) 9 (4) 8

Ans. (3)

Sol. Let $x^2 - 10x + 85 = \lambda$

$\therefore$ Domain for first term

$$\lambda > 0 \quad \dots(1)$$

$$\& 7 - \log_2 \lambda > 0 \Rightarrow \lambda < 2^7 \quad \dots(2)$$

$$\& \log_5 (7 - \log_2 \lambda) > 0 \Rightarrow \lambda < 2^6 \quad \dots(3)$$

$\therefore$ from (1), (2) & (3)

$$0 < \lambda < 2^6$$

$$0 < x^2 - 10x + 85 < 64$$

$$\Rightarrow x \in (3, 7) \quad \dots(A)$$

$$\& \text{domain for second term } -1 \leq \frac{3x-7}{x-17} \leq 1$$

$$\Rightarrow x \in [-5, 6] \quad \dots(B)$$

From (A) & (B), domain of function will be (3, 6]

$$\Rightarrow \alpha = 3, \beta = 6$$

$$\Rightarrow \alpha + \beta = 9$$

15. Let $[\bullet]$ denote the greatest integer function, and let

$f(x) = \min \{ \sqrt{2}x, x^2 \}$. Let $S = \{x \in (-2, 2) : \text{the function } g(x) = |x|[\bullet x^2] \text{ is discontinuous at } x\}$.

Then $\sum_{x \in S} f(x)$ equals :

- (1) $2 - \sqrt{2}$ (2) $2\sqrt{6} - 3\sqrt{2}$
(3) $1 - \sqrt{2}$ (4) $\sqrt{6} - 2\sqrt{2}$

Ans. (3)

Sol. $g(x) = |x|[\bullet x^2]$

points of discontinuity of $g(x)$ in $(-2, 2)$ are

$$(\pm 1, \pm \sqrt{2}, \pm \sqrt{3})$$

$$\therefore S = \{-1, 1, -\sqrt{2}, \sqrt{2}, -\sqrt{3}, \sqrt{3}\}$$

$$\therefore f(x) = \min \{ \sqrt{2}x, x^2 \}$$

$$\therefore \sum_{x \in S} f(x) = -\sqrt{2} + 1 - 2 + 2 - \sqrt{6} + \sqrt{6}$$

$$= 1 - \sqrt{2}$$

16. Let S and S' be the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$

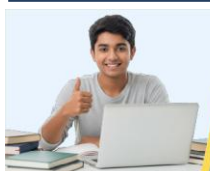
and $P(\alpha, \beta)$ be a point on the ellipse in the first quadrant. If $(SP)^2 + (S'P)^2 - SP \cdot S'P = 37$,

then $\alpha^2 + \beta^2$ is equal to :

- (1) 15 (2) 11
(3) 17 (4) 13

Ans. (4)

Sol. $\therefore P$ lies on ellipse $\Rightarrow \frac{\alpha^2}{25} + \frac{\beta^2}{9} = 1$



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$$\because PS + PS' = 2a \Rightarrow PS + PS' = 10$$

$$\therefore (PS)^2 + (PS')^2 - PS \cdot PS' = 37$$

$$(PS + PS')^2 - 3PS \cdot PS' = 37$$

$$100 - 3PS \cdot PS' = 37$$

$$3PS \cdot PS' = 63 \Rightarrow PS \cdot PS' = 21$$

$$\because PS \text{ \& } PS' \text{ are } \left(5 \pm \frac{4}{5} \cdot \alpha \right)$$

$$\therefore PS \cdot PS' = 25 - \frac{16}{25} \alpha^2 = 21$$

$$\frac{16}{25} \alpha^2 = 4$$

$$\alpha = \frac{5}{2} \Rightarrow \alpha^2 = \frac{25}{4}$$

$$\therefore \beta^2 = \frac{27}{4}$$

$$\therefore \alpha^2 + \beta^2 = \frac{52}{4} = 13$$

17. Let the locus of the mid-point of the chord through the origin O of the parabola $y^2 = 4x$ be the curve S. Let P be any point on S. Then the locus of the point, which internally divides OP in the ratio 3 : 1, is :

- (1) $3y^2 = 2x$ (2) $2y^2 = 3x$
(3) $3x^2 = 2y$ (4) $2x^2 = 3y$

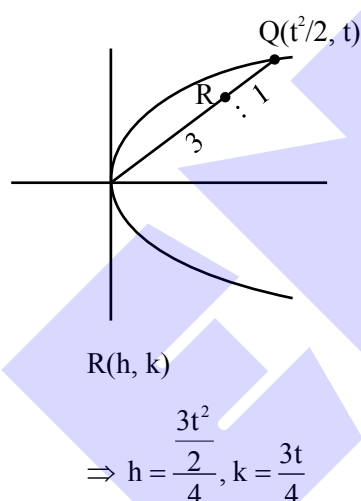
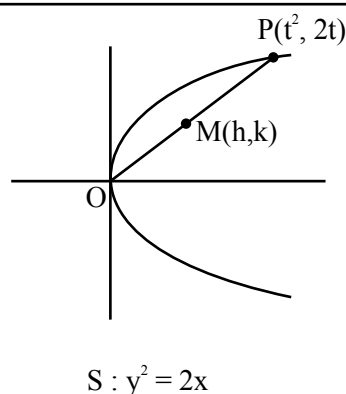
Ans. (2)

Sol. $y^2 = 4x$

Locus of mid point of OP

$$M(h, k) \Rightarrow h = \frac{t^2}{2}, k = t$$

$$\Rightarrow k^2 = 2h \Rightarrow y^2 = 2x$$



$$t^2 = \frac{8h}{3}, t = \frac{4k}{3}$$

$$\Rightarrow \frac{16k^2}{9} = \frac{8h}{3} \Rightarrow 2k^2 = 3h$$

Locus of R : $2y^2 = 3x$

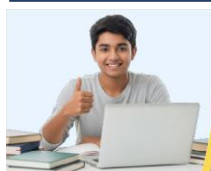
18. Let $f(x) = [x]^2 - [x + 3] - 3$, $x \in \mathbb{R}$ where $[\bullet]$ is the greatest integer function. Then

- (1) $f(x) > 0$ only for $x \in [4, \infty)$
(2) $f(x) < 0$ only for $x \in [-1, 3)$
(3) $\int_0^2 f(x) dx = -6$
(4) $f(x) = 0$ for finitely many values of x .

Ans. (2)

Sol. $f(x) = [x]^2 - [x] - 6 = ([x] + 2)([x] - 3)$

- (1) $f(x) > 0 \Rightarrow [x] \in (-\infty, -2) \cup (3, \infty)$
 $\Rightarrow x \in (-\infty, -2) \cup [4, \infty)$
(2) $f(x) < 0 \Rightarrow [x] \in (-2, 3)$



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$$\Rightarrow x \in [-1, 3)$$

option (2) is correct

$$(3) \int_0^2 f(x) dx = \int_0^1 (0 - 0 - 6) dx + \int_1^2 (1 - 1 - 6) dx$$

$$= -6 - 6$$

$$= -12$$

$$(4) f(x) = 0 \Rightarrow [x] = 3 \text{ or } [x] = -2$$

infinitely many solutions

19. Let f and g be functions satisfying $f(x+y) = f(x)f(y)$, $f(1) = 7$ and $g(x+y) = g(xy)$, $g(1) = 1$, for all $x, y \in \mathbb{N}$. $\sum_{x=1}^n \left(\frac{f(x)}{g(x)} \right) = 19607$, then

n is equal to :

- (1) 7 (2) 5
(3) 6 (4) 4

Ans. (2)

$$\text{Sol. } f(x+y) = f(x)f(y) \Rightarrow f(x) = a^x$$

$$(\because f(1) = 7 \Rightarrow a^1 = 7)$$

$$\text{So } f(x) = 7^x$$

Now

$$g(x+y) = g(xy) \text{ (put } y = 1)$$

$$\Rightarrow g(x+1) = g(x)$$

$$\text{so } g(1) = g(2) = g(3) = \dots = g(n) = 1$$

$$\text{Given } \sum_{x=1}^n \frac{f(x)}{g(x)} = 19607$$

$$\sum_{x=1}^n \frac{7^x}{1} = 19607$$

$$\Rightarrow 7 \left(\frac{7^n - 1}{7 - 1} \right) = 19607$$

$$7^n - 1 = \frac{6}{7} \times 19607$$

$$7^n = 16807 \Rightarrow n = 5$$

20. Let C_r denote the coefficient of x^r in the binomial expansion of $(1+x)^n$, $n \in \mathbb{N}$, $0 \leq r \leq n$.

$$\text{If } P_n = C_0 - C_1 + \frac{2^2}{3} C_2 - \frac{2^3}{4} C_3 + \dots + \frac{(-2)^n}{n+1} C_n,$$

then the value of $\sum_{n=1}^{25} \frac{1}{P_{2n}}$ equals.

- (1) 580 (2) 525

$$(3) 650$$

$$(4) 675$$

Ans. (4)

$$\text{Sol. } P_n = \sum_{r=0}^n \frac{{}^nC_r (-2)^r}{r+1} = \sum_{r=0}^n \frac{1}{(n+1)} {}^{n+1}C_{r+1} (-2)^r$$

$$= \frac{-1}{2(n+1)} \sum_{r=0}^n {}^{n+1}C_{r+1} (-2)^{r+1}$$

$$= \frac{-1}{2(n+1)} [(1-2)^{n+1} - 1]$$

$$P_n = \frac{1}{2(n+1)} [1 - (-1)^{n+1}]$$

$$P_{2n} = \frac{1}{2(2n+1)} [1 - (-1)^{2n+1}]$$

$$P_{2n} = \frac{1}{2n+1}$$

$$\sum_{n=1}^{25} \frac{1}{P_{2n}} = \sum_{n=1}^{25} (2n+1)$$

$$= 3 + 5 + \dots + 51$$

$$= \frac{25}{2} [51 + 3]$$

$$= 25 \times 27 = 675$$

SECTION-B

21. Let a vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \lambda\hat{k}$, $\lambda > 0$, make an obtuse angle with the vector $\vec{b} = -\lambda^2\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$ and an angle θ , $\frac{\pi}{6} < \theta <$

$\frac{\pi}{2}$, with the positive z-axis. If the set of all possible values of λ is $(\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to _____.

Ans. (5)

$$\text{Sol. } \frac{\vec{a} \cdot \hat{k}}{|\vec{a}|} = \cos \theta \Rightarrow \frac{\lambda}{\sqrt{3+\lambda^2}} = \cos \theta$$

$$\Rightarrow 0 < \frac{\lambda}{\sqrt{3+\lambda^2}} < \frac{\sqrt{3}}{2}$$

$$\Rightarrow \lambda > 0 \text{ \& } 4\lambda^2 < 9 + 3\lambda^2 \Rightarrow \lambda^2 < 9$$

$$\Rightarrow \lambda \in (0, 3) \dots (1)$$



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$$\Rightarrow \vec{a} \cdot \vec{b} < 0 \Rightarrow -\sqrt{2}\lambda^2 - 4\sqrt{2} + 4\sqrt{2}\lambda < 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 4 > 0 \Rightarrow (\lambda - 2)^2 > 0$$

$$\Rightarrow \lambda \neq 2 \quad \dots(2)$$

from (1) & (2)

$$\lambda \in (0, 3) - \{2\}$$

$$\therefore \alpha = 0, \beta = 3, \gamma = 2$$

$$\Rightarrow \alpha + \beta + \gamma = 5$$

22. Let $[\bullet]$ be the greatest integer function. If

$$\alpha = \int_0^{64} \left(x^{1/3} - [x^{1/3}] \right) dx, \text{ then}$$

$$\frac{1}{\pi} \int_0^{\alpha\pi} \left(\frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} \right) d\theta \text{ is equal to } \underline{\hspace{2cm}}.$$

Ans. (36)

Sol. $\therefore \int_0^{64} x^{1/3} dx = \frac{3}{4} \left[x^{4/3} \right]_0^{64} = 192$ &

$$\int_0^{64} [x^{1/3}] dx = \int_0^1 [x^{1/3}] dx + \int_1^8 [x^{1/3}] dx + \int_8^{27} [x^{1/3}] dx + \int_{27}^{64} [x^{1/3}] dx = 156$$

$$\text{So } \alpha = 192 - 156 = 36$$

$$\text{Now } E = \frac{1}{\pi} \int_0^{36\pi} \frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta$$

$$= \frac{36}{\pi} \int_0^{\pi} \frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta$$

$$\Rightarrow E = \frac{36 \cdot 2}{\pi} \int_0^{\pi/2} \frac{\sin^2 \theta d\theta}{\sin^6 \theta + \cos^6 \theta}$$

$$\text{Let } J = \int_0^{\pi/2} \frac{\sin^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta \quad \dots(1)$$

Applying King

$$J = \int_0^{\pi/2} \frac{\cos^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta \quad \dots(2)$$

$$\text{Now } 2J = \int_0^{\pi/2} \frac{1}{\sin^6 \theta + \cos^6 \theta} d\theta \quad (\text{add (1) \& (2)})$$

$$= \int_0^{\pi/2} \frac{\sec^6 \theta}{\tan^6 \theta + 1} d\theta$$

$$= \int_0^{\infty} \frac{(1 + \lambda^2)}{\lambda^4 - \lambda^2 + 1} d\lambda$$

$$= \int_0^{\infty} \frac{1 + \frac{1}{\lambda^2}}{\lambda^2 - 1 + \frac{1}{\lambda^2}} d\lambda$$

$$= \pi$$

$$\Rightarrow J = \frac{\pi}{2}$$

$$\Rightarrow E = \frac{36 \cdot 2}{\pi} \times J = 36$$

23. Let $\cos(\alpha + \beta) = -\frac{1}{10}$ and $\sin(\alpha - \beta) = \frac{3}{8}$, where

$$0 < \alpha < \frac{\pi}{3} \text{ and } 0 < \beta < \frac{\pi}{4}.$$

$$\text{If } \tan 2\alpha = \frac{3(1-r\sqrt{5})}{\sqrt{11}(s+\sqrt{5})}, r, s \in \mathbb{N}, \text{ then } r + s \text{ is}$$

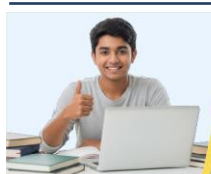
equal to _____.

Ans. (20)

Sol. $\tan 2\alpha = \tan [(\alpha + \beta) + (\alpha - \beta)]$

$$\tan 2\alpha = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)}$$

$$\tan 2\alpha = \frac{\left(-\sqrt{99} + \frac{3}{\sqrt{55}} \right)}{1 - \left(\sqrt{99} \right) \left(\frac{3}{\sqrt{55}} \right)}$$



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$$\tan 2\alpha = \frac{-3\sqrt{11} + \frac{3}{\sqrt{5} \times \sqrt{11}}}{1 + \frac{9\sqrt{11}}{\sqrt{5} \times \sqrt{11}}}$$

$$\tan 2\alpha = \frac{3(1-11\sqrt{5})}{\sqrt{11}(9+\sqrt{5})}$$

$$r = 11, s = 9$$

$$r + s = 20$$

24. Suppose a, b, c are in A.P. and $a^2, 2b^2, c^2$ are in G.P. If $a < b < c$ and $a + b + c = 1$, then $9(a^2 + b^2 + c^2)$ is equal to _____.

Ans. (9)

Sol. $a = b - d, c = b + d, \Rightarrow b = \frac{1}{3}$

$$\Rightarrow 4b^4 = a^2c^2$$

$$\Rightarrow 4b^4 = [(b-d)(b+d)]^2$$

$$\Rightarrow \frac{4}{81} = \left(\frac{1}{9} - d^2\right)^2$$

$$\Rightarrow \left(\frac{1}{9} - d^2\right) = \pm \frac{2}{9}$$

$$d^2 = 1/3 \Rightarrow d = +\frac{1}{\sqrt{3}} \text{ (as } a < b < c)$$

$$\therefore 9(a^2 + b^2 + c^2)$$

$$= 9 \left[\left(\frac{1}{3} - \frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3} + \frac{1}{\sqrt{3}}\right)^2 \right]$$

$$= 9 \left[\frac{1}{3} + \frac{2}{3} \right] = 3 + 6 = 9$$

25. Let S be the set of the first 11 natural numbers. Then the number of elements in $A = \{B \subseteq S : n(B) \geq 2 \text{ and the product of all elements of } B \text{ is even}\}$ is _____.

Ans. (1979)

Sol. $A = \{1, 2, 3, \dots, 11\}$

$\therefore n(B) \geq 2$ & product of all elements in B is even

Case (i) $n(B) = 2 \Rightarrow {}^{11}C_2 - {}^6C_2$

$n(B) = 3 \Rightarrow {}^{11}C_3 - {}^6C_3$

$n(B) = 4 \Rightarrow {}^{11}C_4 - {}^6C_4$

$n(B) = 5 \Rightarrow {}^{11}C_5 - {}^6C_5$

$n(B) = 6 \Rightarrow {}^{11}C_6 - {}^6C_6$

$n(B) = 7 \Rightarrow {}^{11}C_7$

:

:

$n(B) = 11 \Rightarrow {}^{11}C_{11}$

$\therefore \text{number of set } B \Rightarrow \sum_{r=2}^{11} {}^{11}C_r - \sum_{r=2}^6 {}^6C_r$

$= 2^{11} - (12) - (2^6 - 7)$

$= 2048 - 64 - 5$

$= 1979$

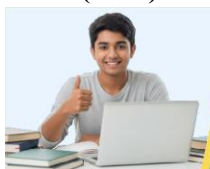
Alternate Solution :

Total subsets $= 2^{11}$

No. of subsets having odd terms only $= 2^6$

No. of subsets having one term only & also having even terms $= 5$

Req. ways $= 2^{11} - 2^6 - 5 = 1979$



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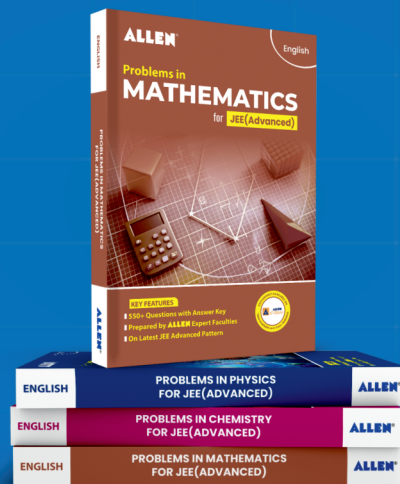
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